# Systems of Equations Cheat Sheet

Three methods to solve systems of two equations with two unknowns, with worked examples.

## Method 1: Graphing

### Graphing Method

1. Rewrite both equations in slope-intercept form ($y = mx + b$)
2. Graph both lines on the same coordinate plane
3. The intersection point $(x, y)$ is the solution

Best for: visualizing solutions, estimating answers. Less precise for non-integer solutions.

## Method 2: Substitution

### Substitution Method

**Example: $y = 2x + 1$ and $3x + y = 11$**

1. Solve one equation for one variable (already done: $y = 2x + 1$)
2. Substitute into the other: $3x + (2x + 1) = 11$
3. Simplify: $5x + 1 = 11$, so $5x = 10$, $x = 2$
4. Back-substitute: $y = 2(2) + 1 = 5$
5. Solution: $(2, 5)$. Check: $3(2) + 5 = 11$

Best when: one equation is already solved for a variable, or a coefficient is 1.

## Method 3: Elimination

### Elimination Method

**Example: $2x + 3y = 12$ and $4x - 3y = 6$**

1. Align equations vertically (already done)
2. Look for opposite coefficients: $+3y$ and $-3y$ cancel
3. Add equations: $(2x + 4x) + (3y - 3y) = 12 + 6$
4. Simplify: $6x = 18$, so $x = 3$
5. Substitute $x = 3$: $2(3) + 3y = 12$, $3y = 6$, $y = 2$
6. Solution: $(3, 2)$. Check: $4(3) - 3(2) = 6$

If no coefficients cancel: multiply one or both equations to create opposites.

## Special Cases

### No Solution (Parallel Lines)

When: same slope, different y-intercept

**Example:** y = 2x + 3 and y = 2x - 1

Lines never intersect. Elimination gives a false statement like 0 = 4.

### Infinite Solutions (Same Line)

When: equations are multiples of each other

**Example:** y = 2x + 3 and 2y = 4x + 6

Same line, every point is a solution. Elimination gives 0 = 0.

## Matrix Method (Basics)

### Setting Up a Matrix

For: ax + by = e and cx + dy = f

Write as coefficient matrix [a b | e] and [c d | f], then use row operations to reduce.

This extends naturally to 3+ variable systems. You will learn this more in linear algebra.

## Word Problem Strategy

### Setting Up Systems from Word Problems

1. Define variables: Let x = ... and y = ...
2. Write two equations from the given information
3. Solve using substitution or elimination
4. Check: does the answer make sense in context?

**Common setups:**
**Total + relationship:** x + y = total, x = 2y + 3
**Mixture:** 0.05x + 0.10y = total value, x + y = total items
**Distance:** rate1 * time1 = rate2 * time2, with shared constraint

**Which method to use? Graphing for visual understanding, substitution when a variable is isolated, elimination when coefficients align nicely.**