Polynomials Cheat Sheet

Comprehensive reference for polynomial operations, factoring, and division.

Polynomial Basics

Terminology

Term	Definition	Example
Monomial	One term	$5x^2$
Binomial	Two terms	$3x + 7$
Trinomial	Three terms	$x^2 + 5x + 6$
Degree	Highest exponent	$x^3 + 2x$ has degree 3
Leading coeff.	Coeff. of highest degree	In $4x^3 - x$, it is 4
Constant term	Term with no variable	In $x^2 + 3$, it is 3

Operations

Adding and Subtracting

Combine like terms (same variable and exponent).

Add:

$(3x^2 + 2x - 1) + (x^2 - 4x + 5)$
$= 4x^2 - 2x + 4$

Subtract:

$(3x^2 + 2x - 1) - (x^2 - 4x + 5)$
$= 2x^2 + 6x - 6$

Multiplying

FOIL (for two binomials):

$(a + b)(c + d) =$
$ac + ad + bc + bd$

Distribution (general):

Multiply each term of one polynomial by every term of the other, then combine like terms.

Factoring Methods

Always Start With GCF

Factor out the greatest common factor first.

$6x^3 + 9x^2 = 3x^2(2x + 3)$

Factoring by Grouping

For 4 terms: group pairs, factor each.

$x^3 + 3x^2 + 2x + 6 = x^2(x+3) + 2(x+3) = (x^2+2)(x+3)$

Factoring Trinomials ($x^2 + bx + c$)

Find two numbers that multiply to c and add to b.
Example: $x^2 + 7x + 12 = (x + 3)(x + 4)$ because $3 * 4 = 12$, $3 + 4 = 7$

Special Products

Perfect square (sum): $(a + b)^2 = a^2 + 2ab + b^2$
Perfect square (diff): $(a - b)^2 = a^2 - 2ab + b^2$
Difference of squares: $a^2 - b^2 = (a + b)(a - b)$
Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Diff of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Polynomial Long Division

Step-by-Step

1. Divide the leading term of the dividend by the leading term of the divisor
2. Multiply the entire divisor by that result
3. Subtract from the dividend
4. Bring down the next term
5. Repeat until the degree of the remainder is less than the divisor

Example: $(x^2 + 5x + 6) / (x + 2) = x + 3$ remainder 0

Remainder Theorem

Theorem: When polynomial f(x) is divided by (x - a), the remainder equals f(a).

Factor Theorem: If f(a) = 0, then (x - a) is a factor of f(x).

Example: $f(x) = x^2 + 5x + 6$. $f(-2) = 4 - 10 + 6 = 0$, so (x + 2) is a factor.

Factoring order: GCF first, then check for special patterns (difference of squares, perfect square trinomial), then try grouping or the AC method.