

# Limits Cheat Sheet

Key limit concepts, evaluation techniques, and important results for calculus.

## Definition & Notation

**Notation:**  $\lim_{x \rightarrow a} f(x) = L$

**Meaning:** As  $x$  approaches  $a$ ,  $f(x)$  approaches  $L$

**Left-hand:**  $\lim_{x \rightarrow a^-} f(x)$  (approach from left)

**Right-hand:**  $\lim_{x \rightarrow a^+} f(x)$  (approach from right)

**Exists when:** Left-hand limit = Right-hand limit

## Evaluation Methods

**1. Direct substitution:** Plug in  $x = a$ . If you get a number, that is the limit.

**2. Factoring:** Factor and cancel common terms, then substitute.

**3. Rationalization:** Multiply by conjugate to eliminate radicals.

**4. Simplify:** Combine fractions or expand, then substitute.

## Important Limits

$$\lim_{x \rightarrow 0} \sin(x)/x = 1$$

$$\lim_{x \rightarrow 0} (1 - \cos(x))/x = 0$$

$$\lim_{x \rightarrow \infty} (1 + 1/x)^x = e$$

$$\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$$

## Limits at Infinity

**Constant:**  $\lim_{x \rightarrow \infty} c = c$

**1/x:**  $\lim_{x \rightarrow \infty} 1/x = 0$

**Polynomials:** Compare highest power. Divide all terms by  $x^n$  (highest degree).

**Same degree:** Ratio of leading coefficients

**Higher in numerator:** Limit is infinity

**Higher in denominator:** Limit is 0

## L'Hopital's Rule

**When:** Direct substitution gives  $0/0$  or  $\infty/\infty$

**Rule:**  $\lim f(x)/g(x) = \lim f'(x)/g'(x)$

**Repeat:** Apply again if result is still  $0/0$  or  $\infty/\infty$

**Caution:** Only works for indeterminate forms

## Continuity

**Definition:**  $f$  is continuous at  $x = a$  when all three hold:

- $f(a)$  exists (defined at the point)
- $\lim_{x \rightarrow a} f(x)$  exists (limit exists)
- $\lim_{x \rightarrow a} f(x) = f(a)$  (limit equals value)

**Squeeze theorem:** If  $g(x) \leq f(x) \leq h(x)$  and  $\lim g = \lim h = L$ , then  $\lim f = L$

Always try direct substitution first. If you get  $0/0$ , try factoring or L'Hopital's Rule. If you get a number, you are done.